Banks Lane Junior School

Mathematics Progression Policy

September 2019

This policy sets out the key knowledge and skills pupils should acquire in Primary School in the four operations and fractions, decimals and percentages. It is intended to be used by staff in the planning and teaching of these topics to support sequencing and progression. Staff should use the strategies outlined for their year group for the most part: it is aimed at KS2 but contains the steps in learning in KS1 to support the teaching of low ability and SEND pupils. *If you need to use strategies from the KS1 sections or teach objectives from a lower year group, a discussion with the YGL and/or mathematics lead should have taken place.* It has largely been adapted from the White Rose Maths Hubs Calculations Policy with further material added or removed. It is a working document and will be revised and amended as necessary.

Mrs. S. Scott
Mathematics Lead
Banks Lane Junior School

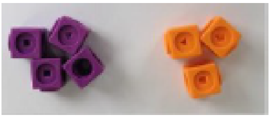
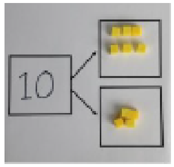
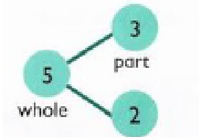
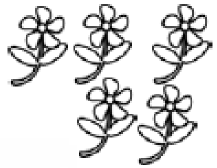
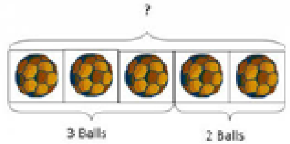
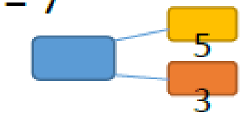
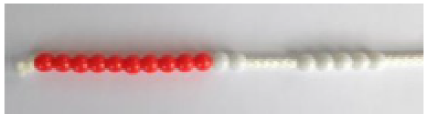
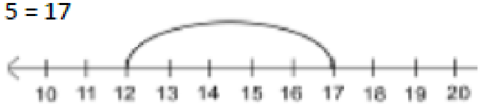
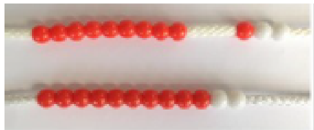
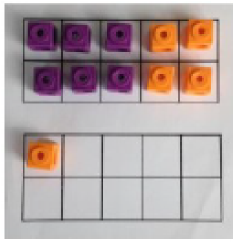
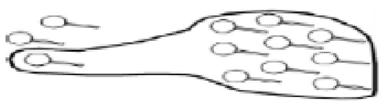
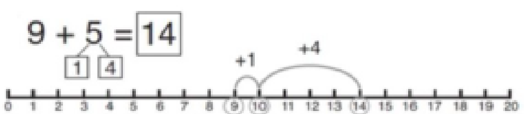
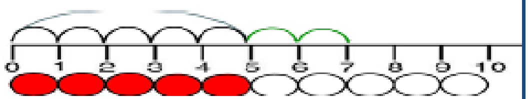
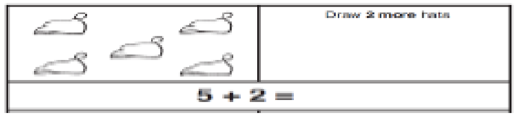
Content

Sections:

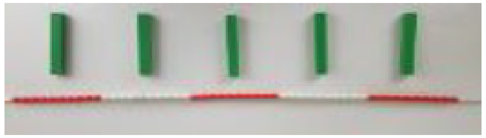
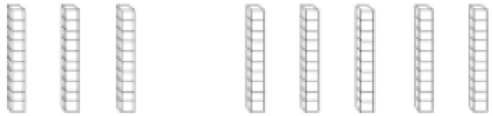
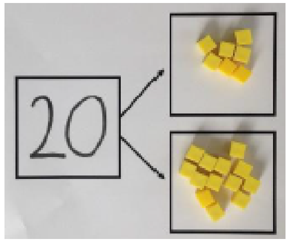
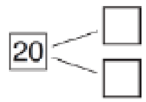
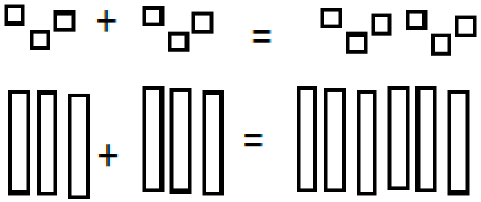
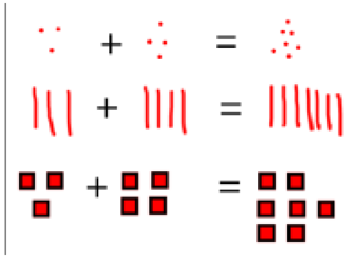
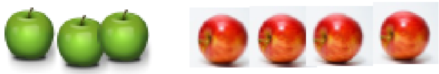
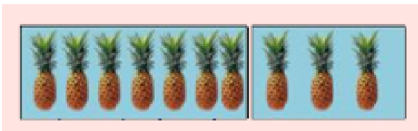
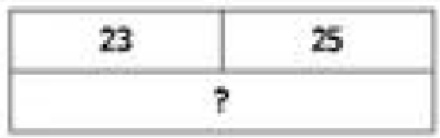
<u>Section 1</u>

Progression in Addition

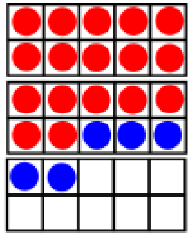
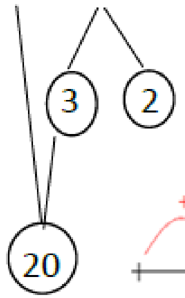
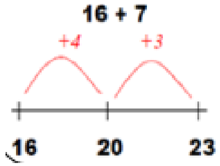
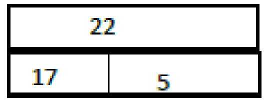
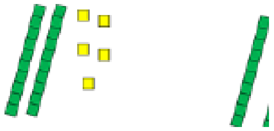
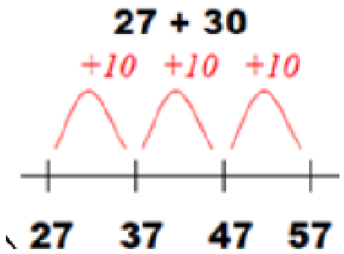
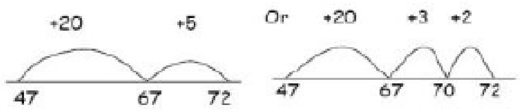
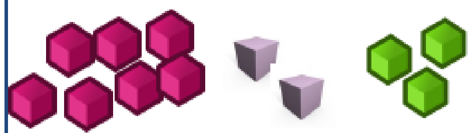
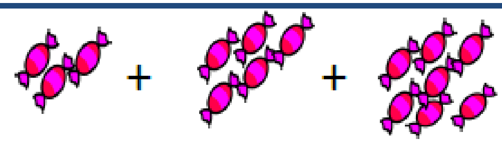
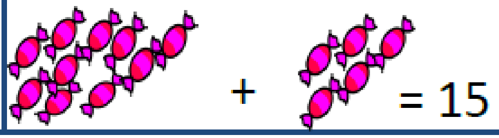
Progression in Addition – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

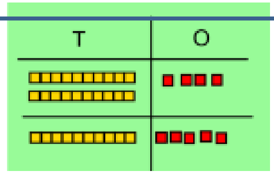
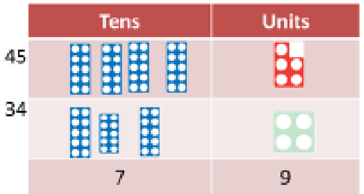
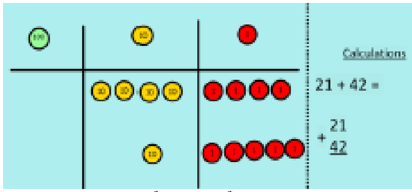
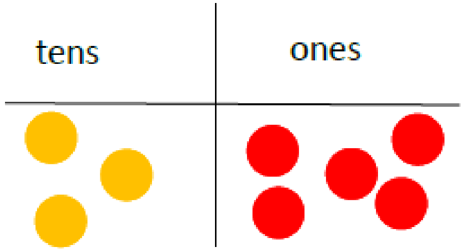
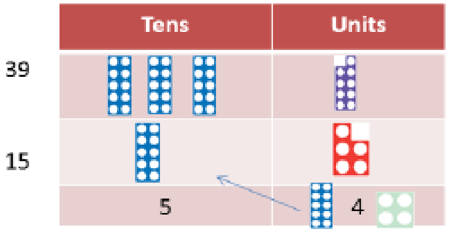
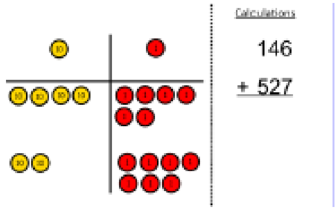
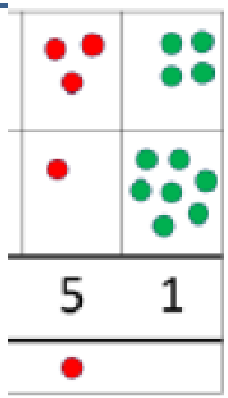
Progression in Addition – Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20  Model using dienes and bead strings	 3 tens + 5 tens = _____ tens 30 + 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 □ + □ = 20 20 – □ = □ □ + □ = 20 20 – □ = □	□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1
Using known facts		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

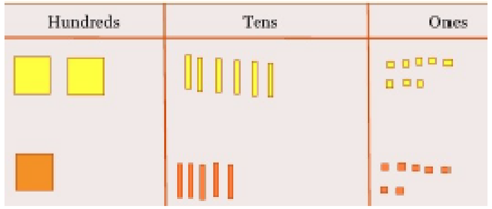
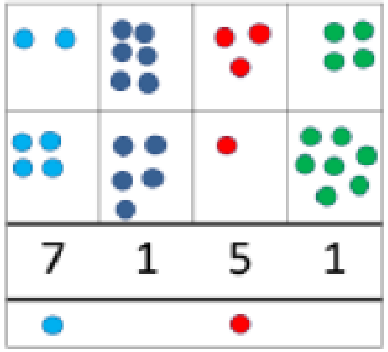
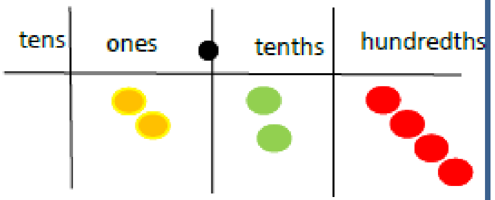
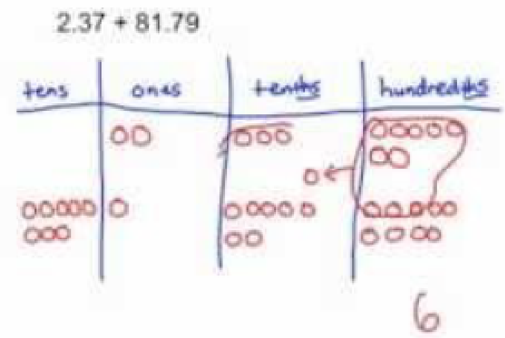
Progression in Addition – Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 17 + 5 = 22   	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens	 25 + 10 = 35 Explore that the ones digit does not change	 	27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation.  	4 + 7 + 6 = $\boxed{10}$ + $\boxed{7}$ 10 = $\boxed{17}$ Combine the two numbers that make/ bridge ten then add on the third.

Progression in Addition – Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	 Model using Dienes or numicon Add together the ones first, then the tens.   Move to using place value counters	Children move to drawing the counters using a tens and one frame. 	$$\begin{array}{r} 2\ 2\ 3 \\ +\ 1\ 1\ 4 \\ \hline 3\ 3\ 7 \end{array}$$ Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 Exchange ten ones for a ten. Model using numicon and pv counters. 	 Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line	$$\begin{array}{lll} 20 & + & 5 \\ 40 & + & 8 \\ \hline 60 & + & 13 \quad = 73 \end{array}$$ Start by partitioning the numbers before formal column to show the exchange. $$\begin{array}{r} 536 \\ +\ 85 \\ \hline 621 \\ \hline {}_{1\ 1} \end{array}$$

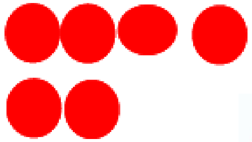
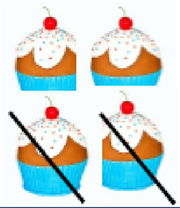
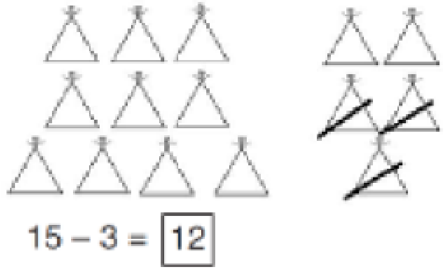
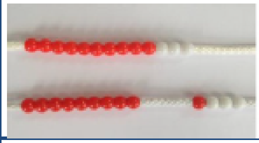
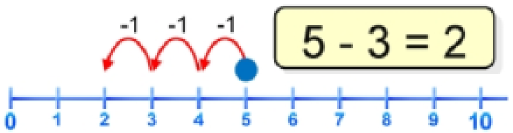
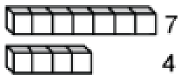
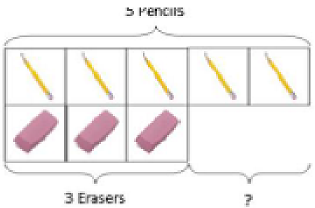
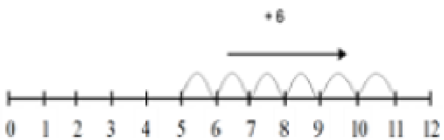
Progression in Addition – Year 4-6

Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using pv grid.	3517 + 396 3913 1 1 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4  Introduce decimal place value counters and model exchange for addition.		72.8 + 54.6 127.4 1 1 £ 23 . 59 + £ 7 . 55 £ 31 . 14
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	81,059 3,668 15,301 + 20,551 120,579 Insert zeros for place holders. 23 . 361 9 . 080 59 . 770 + 1 . 300 93 . 511 2 1 2

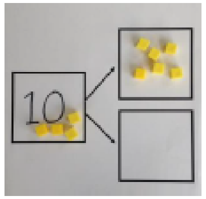
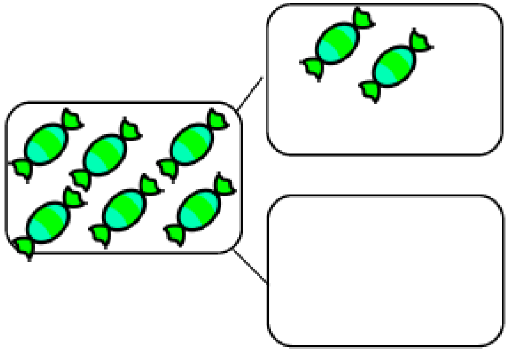
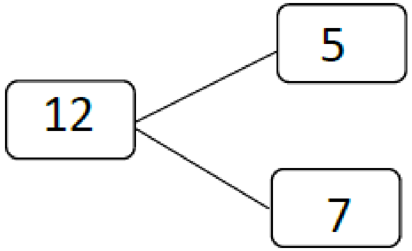
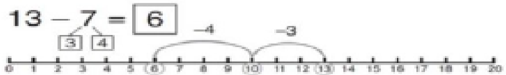
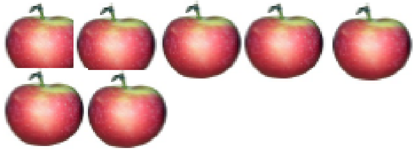
<u>Section 2</u>

Progression in Subtraction

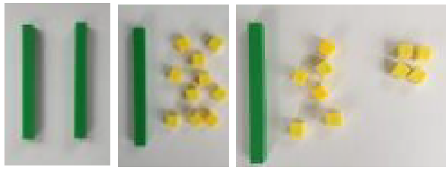
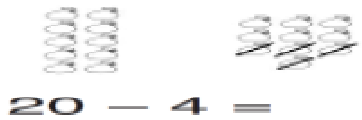
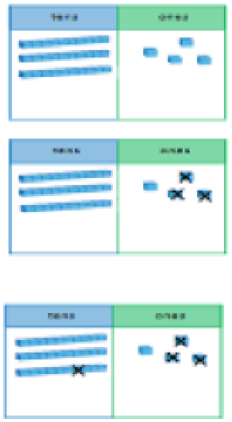
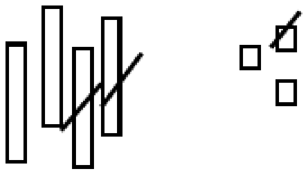
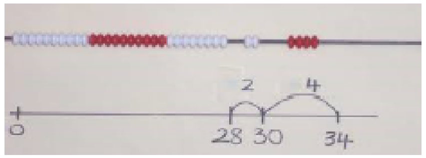
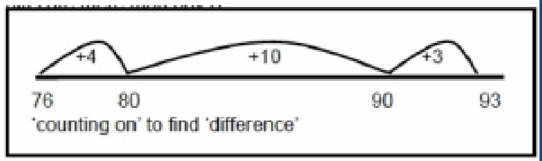
Progression in Subtraction – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2	 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent bar model.	Count on using a number line to find the difference.  	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister.?

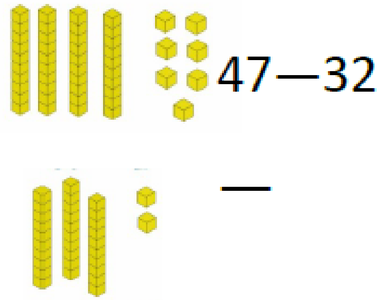
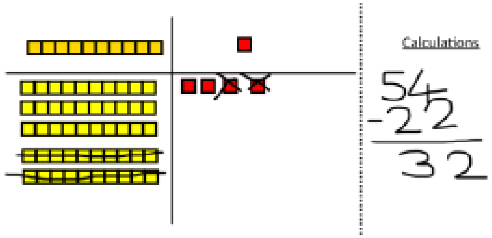
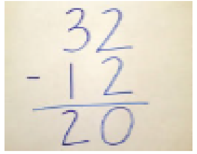
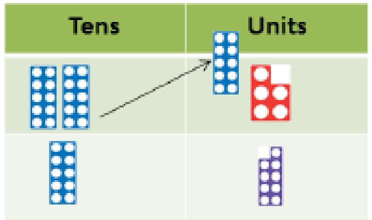
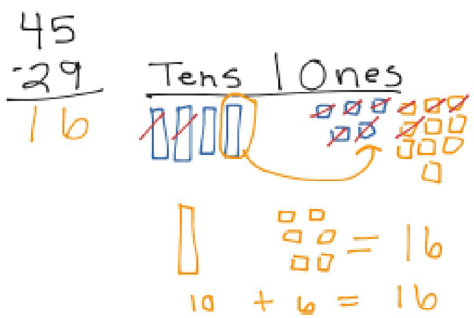
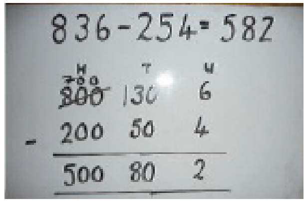
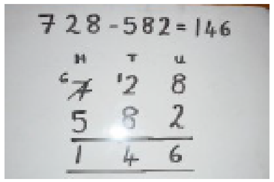
Progression in Subtraction – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	14—9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off?
Bar model	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

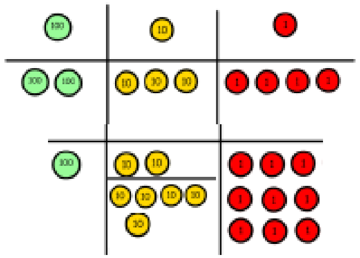
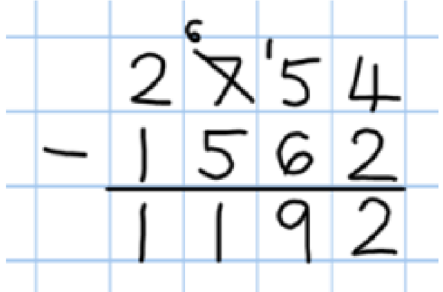
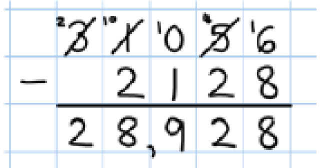
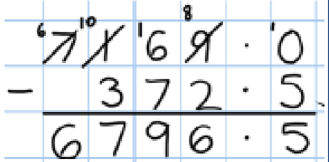
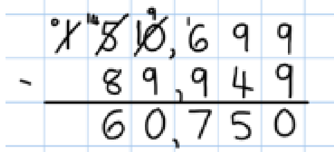
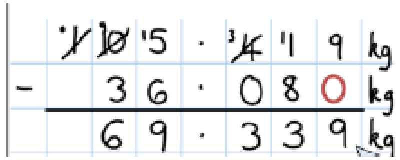
Progression in Subtraction – Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without regrouping. *'Friendly numbers'*	34—13 = 21  Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

Progression in Subtraction – Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Numicon to model	 Draw representations to support understanding	$47-24=23$ $\begin{array}{r} 40+7 \\ -\ 20+4 \\ \hline 20+3 \end{array}$ Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	 Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	 Children may draw base ten or PV counters and cross off.	 Begin by partitioning into pv columns  Then move to formal method.

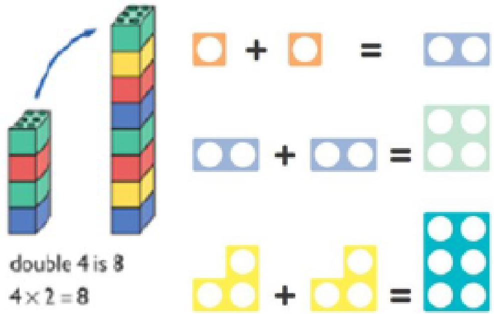
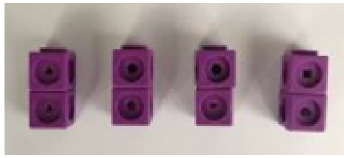
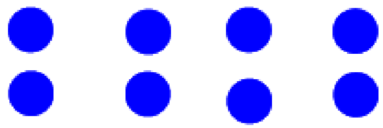
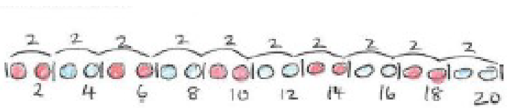
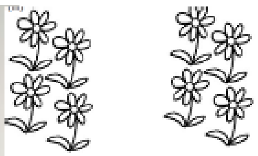
<u>Progression in Subtraction – Year 4-6</u>

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179  Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	 Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	 Use zeros for place-holders. 
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			 

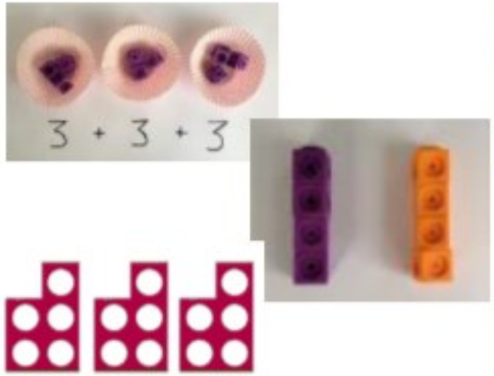
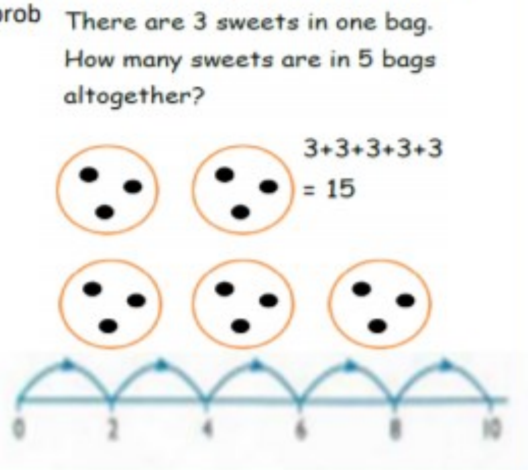
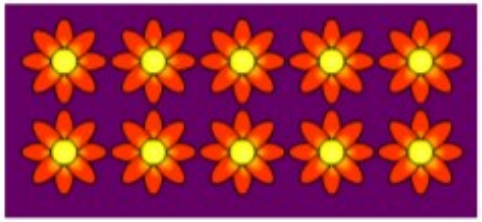
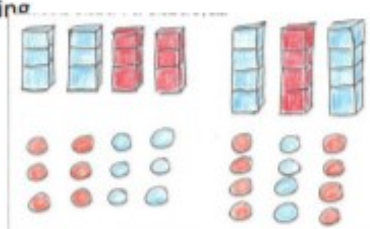
Section 3

Progression in Multiplication

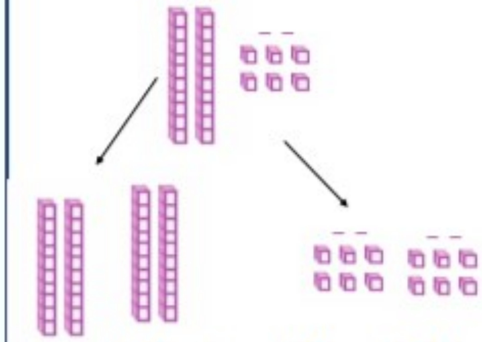
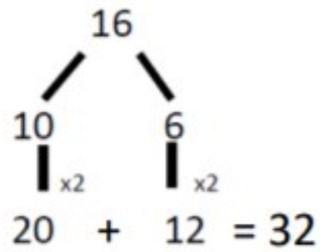
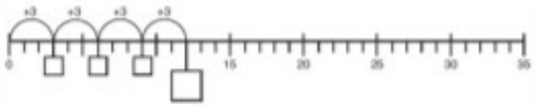
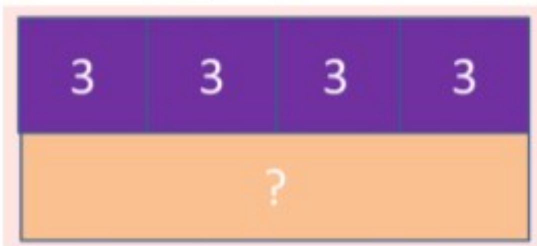
Progression in Multiplication – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers Double 4 is 8 	Partition a number and then double each part before recombining it back together. 16 10 6 x2 x2 20 + 12 = 32
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	  $\square \times \square = 8$ Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8

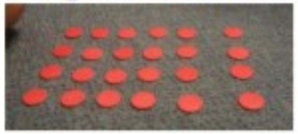
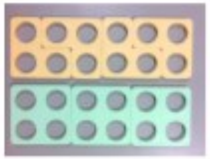
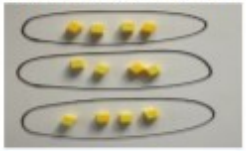
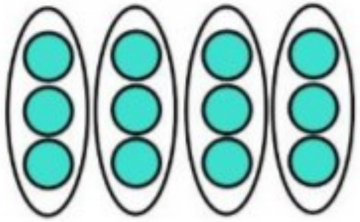
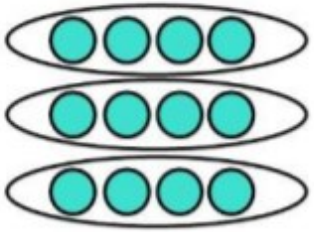
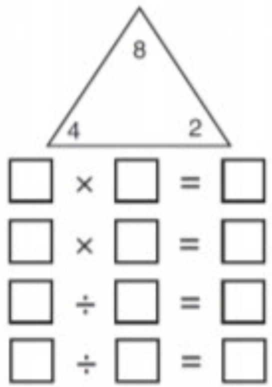
Progression in Multiplication – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	 Use different objects to add equal groups	Use pictorial including number lines to solve prob 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10

Progression in Multiplication – Year 2

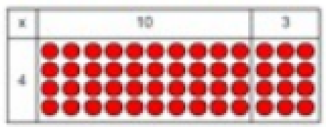
Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.     	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Progression in Multiplication – Year 2

Objective & Strategy	**Concrete**	**Pictorial**	**Abstract**
Multiplication is commutative	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.

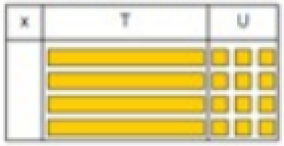
Progression in Multiplication – Year 3

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Grid method</td>
<td>
Show the links with arrays to first introduce the grid method.



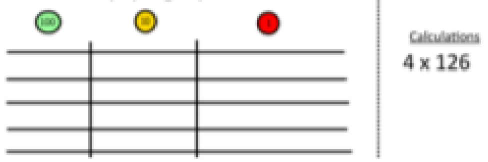
4 rows of 10
4 rows of 3

Move onto base ten to move towards a more compact method.

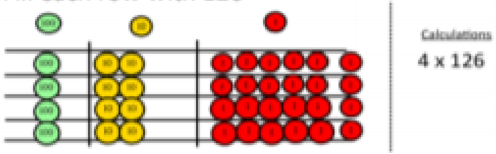


4 rows of 13

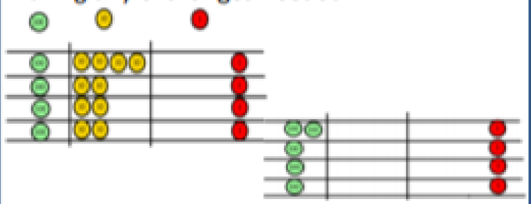
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows



Fill each row with 126

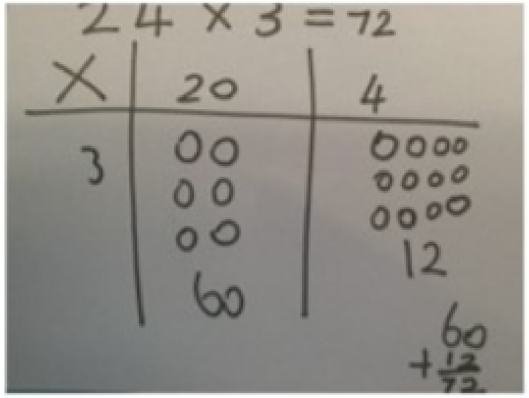


Add up each column, starting with the ones making any exchanges needed

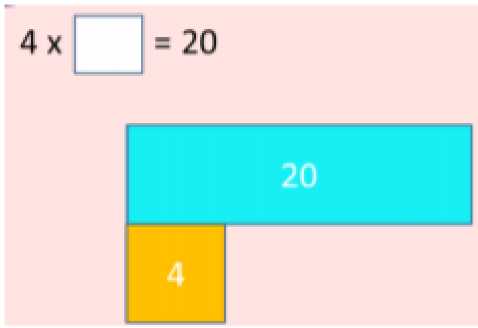


Then you have your answer.
</td>
<td>
Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers



</td>
<td>
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

<table>
<tr><th>×</th><th>30</th><th>5</th></tr>
<tr><td>7</td><td>210</td><td>35</td></tr>
</table>

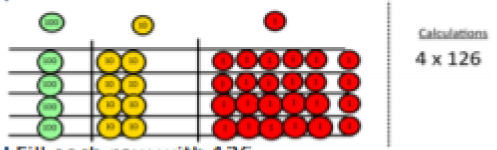
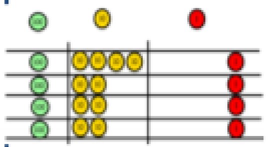
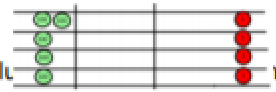
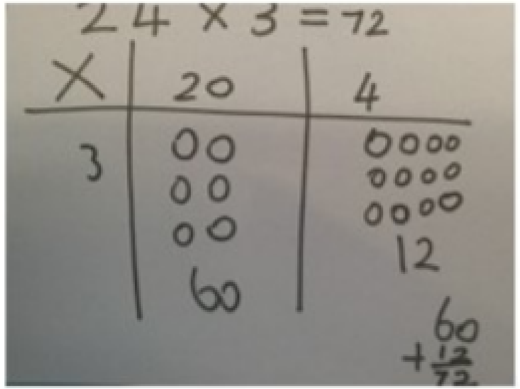
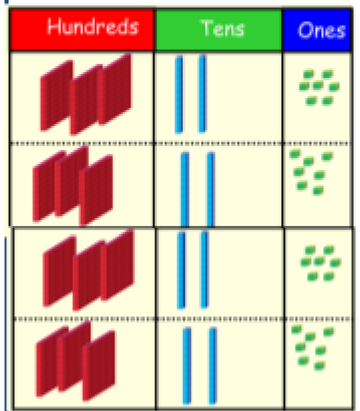
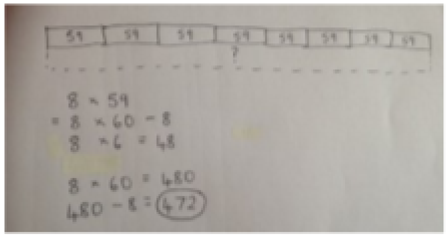
210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

<table>
<tr><th></th><th>10</th><th>8</th></tr>
<tr><td>10</td><td>100</td><td>80</td></tr>
<tr><td>3</td><td>30</td><td>24</td></tr>
</table>
</td>
</tr>
</table>

Progression in Multiplication – Year 4

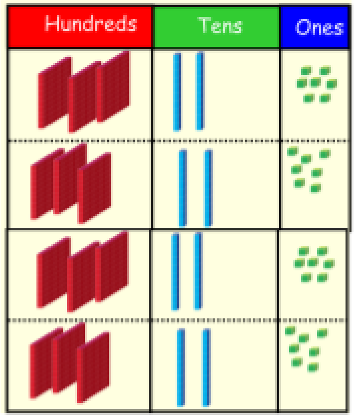
<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Grid method recap from year 3 for 2 digits x 1 digit

Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</td>
<td>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows

Fill each row with 126


Add up each column, starting with the ones making any exchanges needed</td>
<td>Children can represent their work with place value counters in a way that they understand.
They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.
</td>
<td>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
<table><tr><th>x</th><th>30</th><th>5</th></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642

It is important at this stage that they always multiply the ones first.
The corresponding long multiplication is modelled alongside</td>
<td><table><tr><th>x</th><th>300</th><th>20</th><th>7</th></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
The grid method my be used to show how this relates to a formal written method.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</td>
<td>327
x 4
28
80
1200
1308

3 2 7
x 4
1 3 0 8
1 2

This may lead to a compact method.</td>
</tr>
</table>

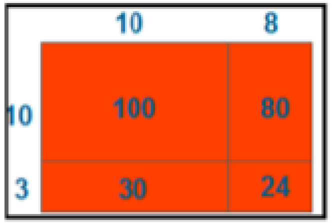
Progression in Multiplication – Year 5-6

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication for 3 and 4 digits x 1 digit.</td>
<td>

It is important at this stage that they always multiply the ones first.

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642</td>
<td><table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table></td>
<td>327
x 4
28
80
1200
1308

3 2 7
× 4
1 3 0 8
1 2

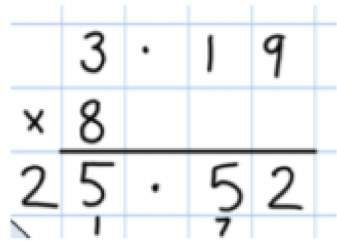
This will lead to a compact method.</td>
</tr>
<tr>
<td>Column multiplication</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</td>
<td>

Continue to use bar modelling to support problem solving</td>
<td>1 8
× 1 3
5 4
2
1 8 0
2 3 4

18 x 3 on the first row
(8 x 3 =24, carrying the 2 for 20, then 1 x 3)
18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first

1 2 3 4
× 1 6
7 4 0 4 (1234 × 6)
1 2 3 4 0 (1234 × 10)
1 9,7 4 4</td>
</tr>
</table>

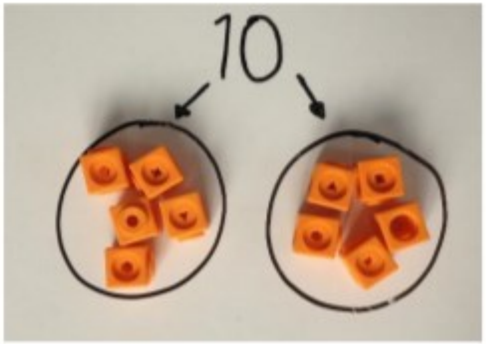
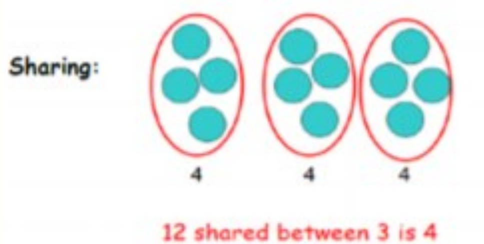
<u>Progression in Multiplication – Year 6</u>

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal plac-es by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

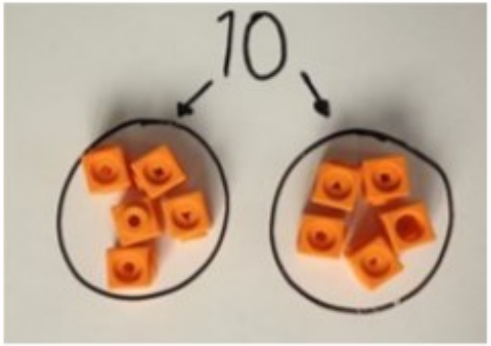
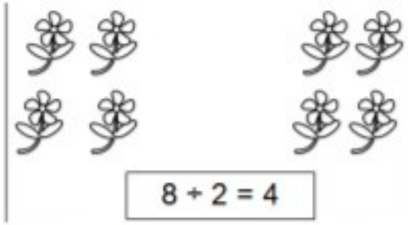
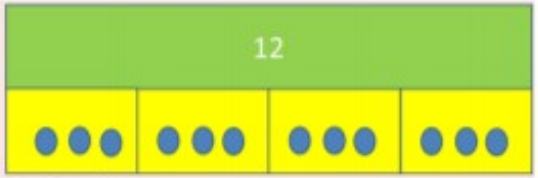
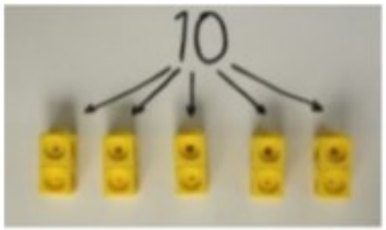
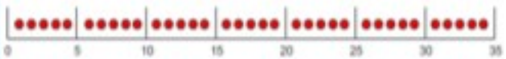
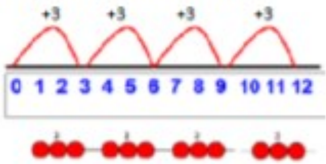
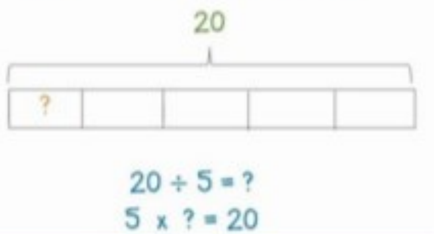
Section 4

Progression in Division

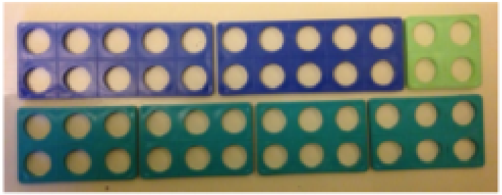
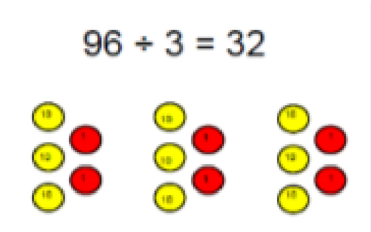
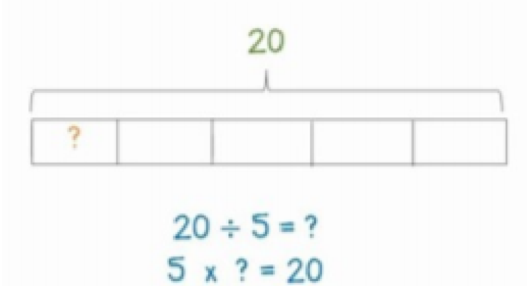
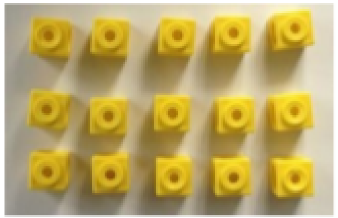
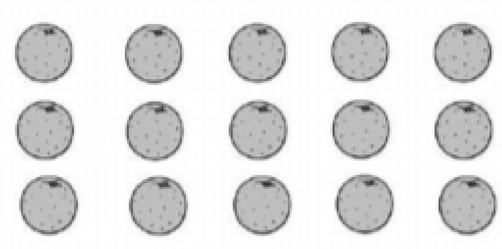
Progression in Division – Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 	12 shared between 3 is 4

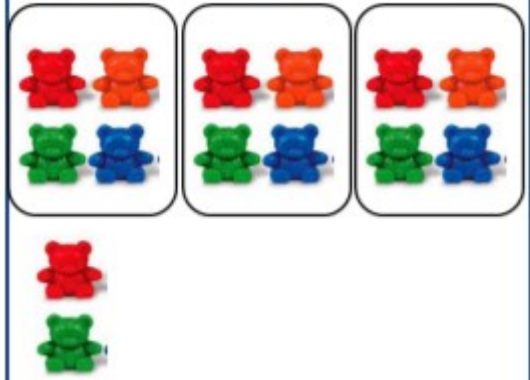
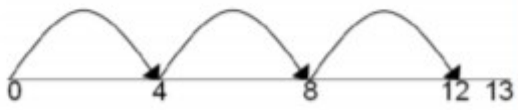
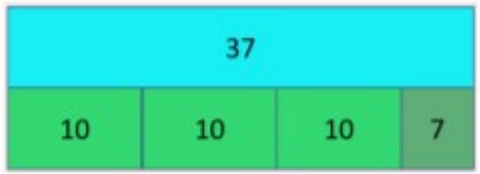
Progression in Division – Year 2

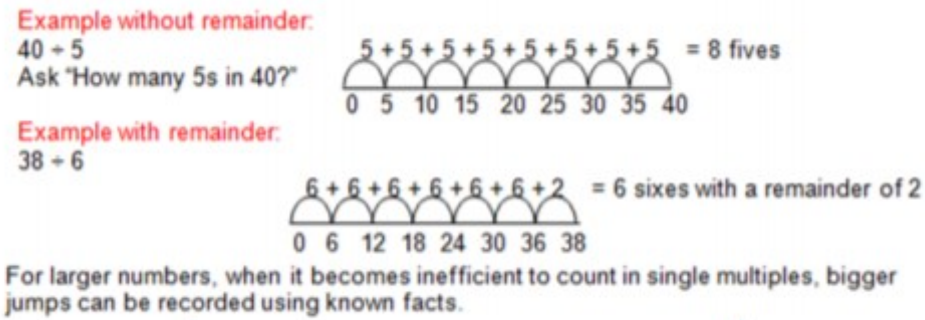
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Progression in Division – Year 3

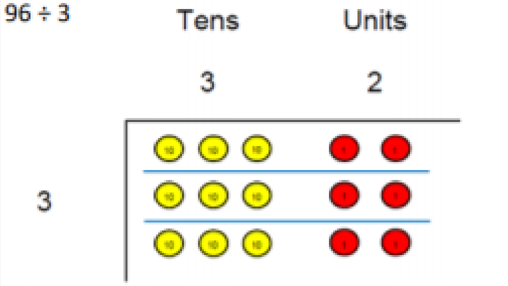
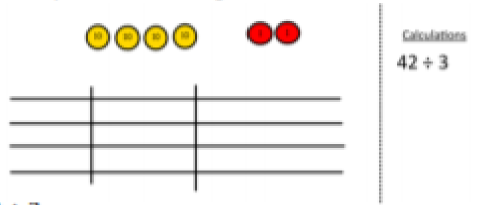
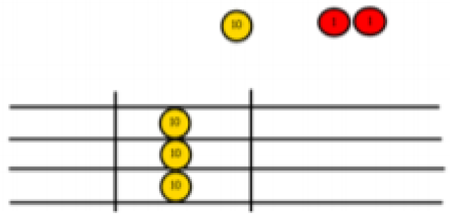
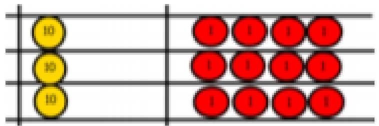
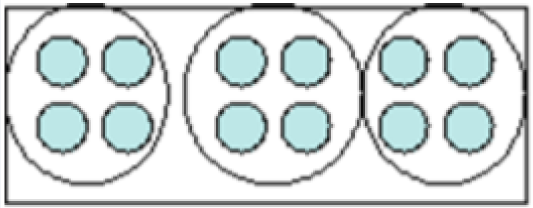
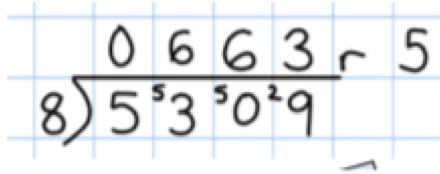
Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4  	Continue to use bar modelling to aid solving division problems.  	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4

Progression in Division – Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders. 	Complete written divisions and show the remainder using r. 



Progression in Division- Year 4-6

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	 Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 2 1 8 4 | 8 ^{3}7 2 Move onto divisions with a remainder. 8 6 r 2 5 | 4 ^{3}3 2 Finally move into decimal places to divide the total accurately. 1 4 . 6 3 5 | 5 ^{16}1 1 . ^{21}0 

Progression in Division- Year 6

Long Division

Step 1- a remainder in the ones

```
     h t o
     0 4 1 R1
    ________
4 )  1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th h t o
     0  4 0 0 R7
    ___________
8 )  3  2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Progression in Division- Year 6

Long Division

Step 1- continued

$$
\begin{array}{rrrrr}
 & & \text{h} & \text{t} & \text{o} \\
 & & 0 & 6 & 1 \\
\cline{3-5}
4 &) & 2 & 4 & 7 \\
 & & & - & 4 \\
\cline{4-5}
 & & & & 3
\end{array}
$$

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

$$
\begin{array}{rrrrrr}
 & & \text{th} & \text{h} & \text{t} & \text{o} \\
 & & 0 & 4 & 0 & 2 \\
\cline{3-6}
4 &) & 1 & 6 & 0 & 9 \\
 & & & & - & 8 \\
\cline{5-6}
 & & & & & 1
\end{array}
$$

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Progression in Division- Year 6

Long Division

Step 1- a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2) 5 8	t o 2 2) 5 8 - 4 1	t o 2 9 2) 5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2) 5 8 - 4 1 8	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0	t o 2 9 2) 5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Progression in Division- Year 6

Long Division

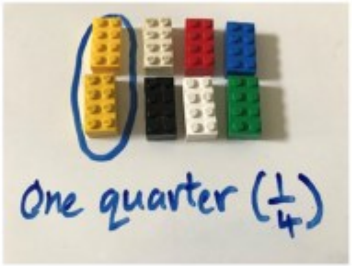
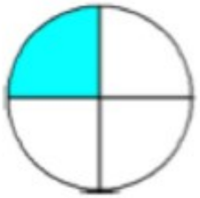
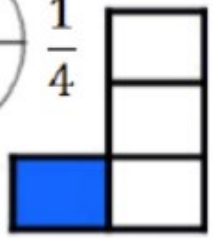
Step 2- a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)278 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)278 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 18 2)278 -2 ↓ 07 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 13 2)278 -2 07 Divide 2 into 7. Place 3 into the quotient.	h t o 13 2)278 -2 07 - 6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 13 2)278 -2 07 - 6 18 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 139 2)278 -2 07 - 6 18 Divide 2 into 18. Place 9 into the quotient.	h t o 139 2)278 -2 07 - 6 18 -18 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 139 2)278 -2 07 - 6 18 -18 0 There are no more digits to drop down. The quotient is 139.

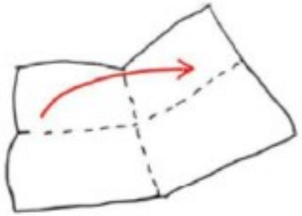
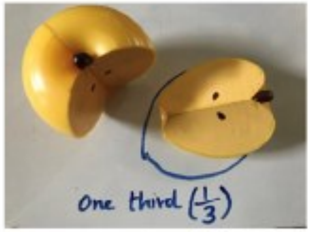
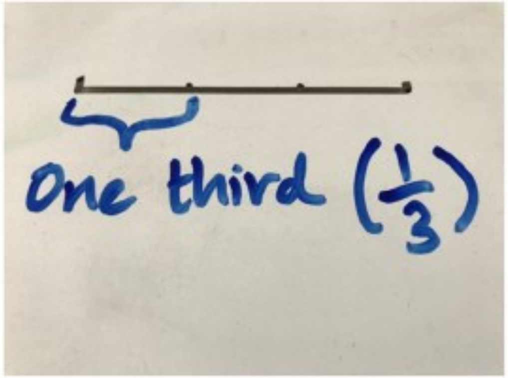
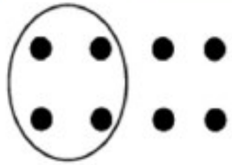
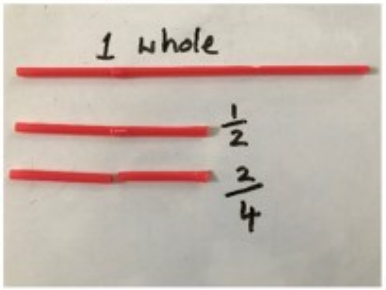
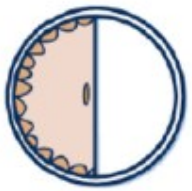
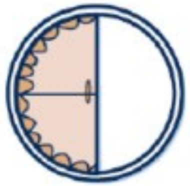
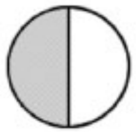
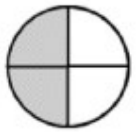
Section 5

Progression in Fractions, Decimals and Percentages

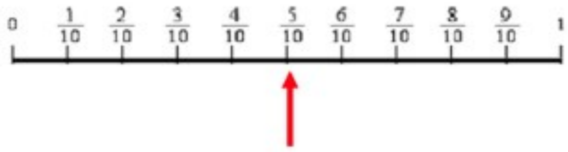
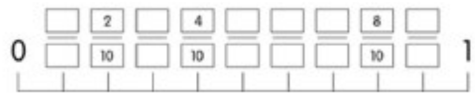
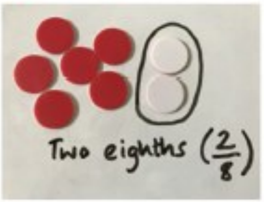
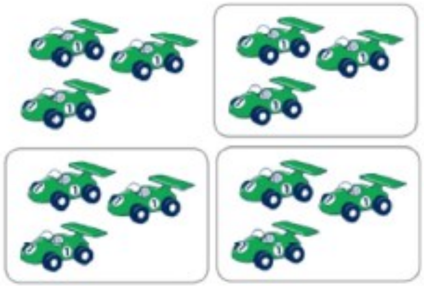
Progression in fractions- Year 1

Concrete	Pictorial	Abstract
Recognise, find and name a half as one of two equal parts of an object, shape or quantity  	A whole cake  1 1 half of the cake  $\frac{1}{2}$	Half of 2 is... ☐ Half of 6 is... ☐ Half of 10 is... ☐ Half of 12 is... ☐
Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity  	 $\frac{1}{4}$ 	A quarter of 4 is... ☐ A quarter of 8 is ☐ A quarter of 12 is ☐ A quarter of 20 is ☐

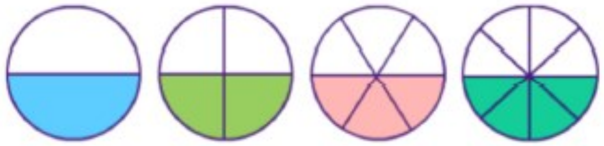
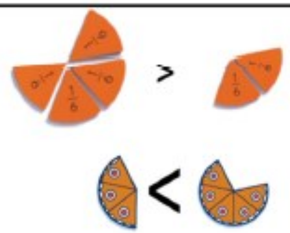
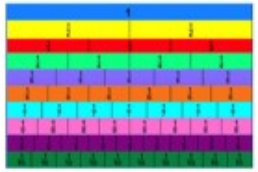
Progression in fractions– Year 2

Concrete	Pictorial	Abstract
Recognise, find, name and write fractions $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ of a length, shape, set of objects or quantity  		$\frac{2}{4}$ of 8 = $\square$ 
Write simple fractions and recognise the equivalence of $\frac{1}{2}$ and $\frac{2}{4}$ 	 I have $\frac{1}{2}$ a pie  You have $\frac{2}{4}$ of a pie	$\frac{1}{2} = \frac{}{4}$  $\frac{}{2}$  $\frac{}{4}$

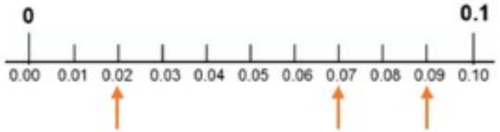
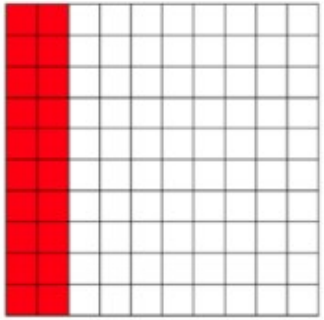
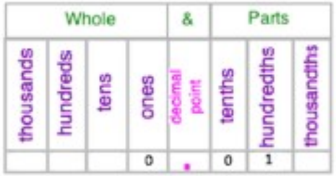
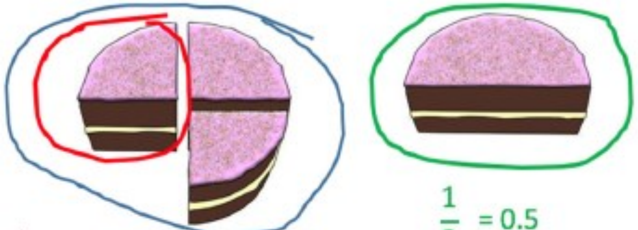
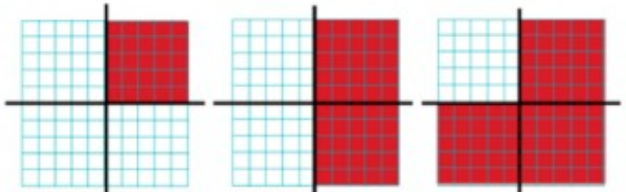
Progression in fractions– Year 3

Concrete	Pictorial	Abstract
Count up and down in tenths: recognise that tenths arise from dividing an object into ten equal parts and in dividing one-digit numbers or quantities by ten  $\frac{1}{2}$	 $\frac{3}{10}$ $\frac{3}{10}$ $\frac{3}{10}$	$\frac{1}{10}$ of 6 is 0.6 because $6 \div 10 = 0.6$ 
Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions and use fractions as numbers   	$\frac{3}{4}$ 	$\frac{1}{5}$ of 15 sweets is 3 because $15 \div 5 = 3$ $\frac{2}{5}$ of 15 sweets is 6 because $15 \div 5 = 3$ and there are two fifths so we multiply the answer by 2 which gives the answer 6

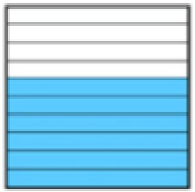
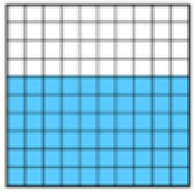
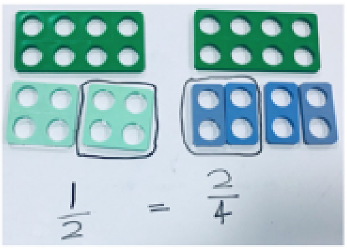
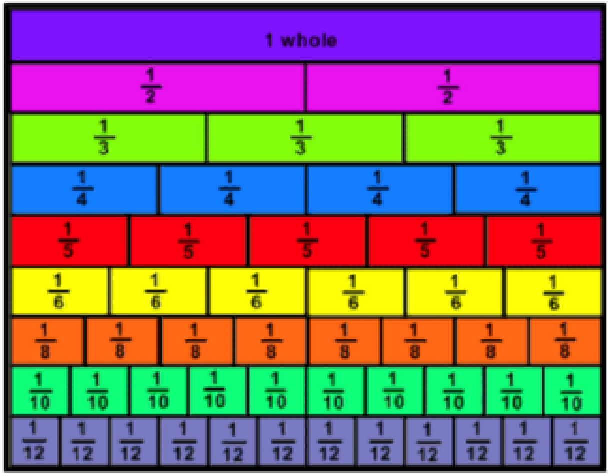
Progression in fractions– Year 3

Concrete	Pictorial	Abstract
Recognise and show, using diagrams equivalent fractions with small denominators  $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$	 $\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$	Sally says that asking for two quarters of a cake is the same as asking for one half? Is she correct? Explain.
Add and subtract fractions with the same denominator  $\frac{1}{2}$ $\frac{1}{2}$ $\frac{2}{2}$ Or 1 whole	 $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$ $\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$	$\frac{1}{5} + \frac{2}{5} = \frac{(1+2)}{5}$ Add the numerators No change to Denominator Common Denominator
Compare and order unit fractions with the same denominators 	  $\frac{1}{6}$ $\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$ $\frac{5}{6}$ $\frac{6}{6}$	$\boxed{\frac{2}{8}}$ $\boxed{\frac{3}{8}}$ $\boxed{\frac{5}{8}}$ $\boxed{\frac{7}{8}}$

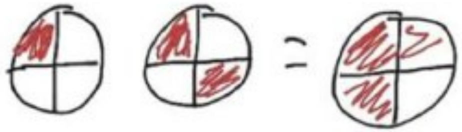
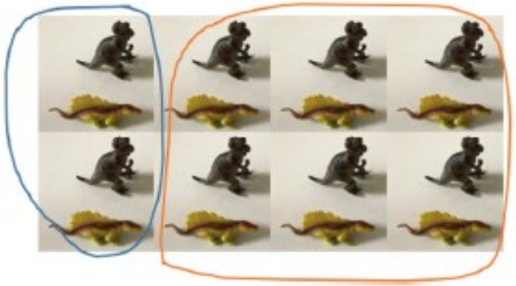
Progression in fractions– Year 4

Concrete	Pictorial	Abstract
Count up and down in hundredths: recognise that hundreds arise when dividing an object by 100 and dividing tenths by 10  $\frac{2}{100}$ $\frac{7}{100}$ $\frac{9}{100}$	 20 hundredths $\frac{20}{100}$  1 hundredth = 0.01 = $\frac{1}{100}$	$\frac{1}{100}$ of 60 = 0.6 because 60 ÷ 100 = 0.6 $\frac{1}{10}$ of 70 = 0.7 so $\frac{1}{100}$ of 70 = 0.07
Recognise and write decimal equivalents to $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$  $\frac{1}{4}$ = 0.25 $\frac{3}{4}$ = 0.75 $\frac{1}{2}$ = 0.5	 $\frac{1}{4}$ = 0.25 $\frac{1}{2}$ = 0.5 $\frac{3}{4}$ = 0.75	$\frac{1}{2}$ = 0.5 $\frac{1}{4}$ = 0.25 $\frac{3}{4}$ = 0.75

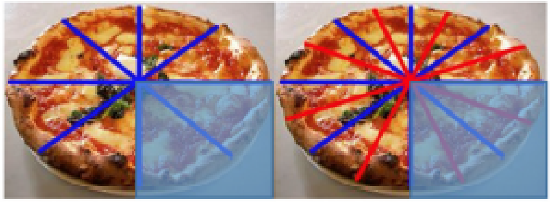
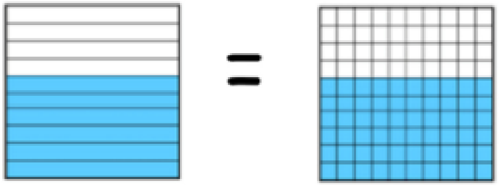
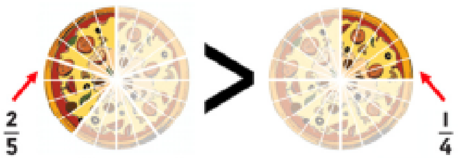
Progression in fractions- Year 4

Concrete	Pictorial	Abstract
Recognise and write decimal equivalents of any number of tenths or hundredths  $\frac{2}{10}$ Or 0.2 of the whole bar	 **0.6** six tenths  **0.60** sixty hundredths	Two tenths is 0.2 which is shown as the fraction $\frac{2}{10}$ $\frac{4}{10} = 0.4$ $\frac{5}{10} = \frac{1}{2} = 0.5$ $\frac{8}{100} = 0.08$
Recognise and show using diagrams, families of common equivalents   Is equal to  	 	$\frac{2}{3} = \frac{4}{6}$ $\frac{3}{5} = \frac{6}{10}$ $\frac{2}{12} = \frac{1}{6}$

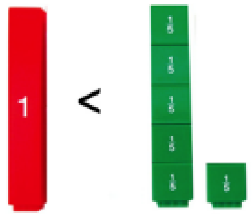
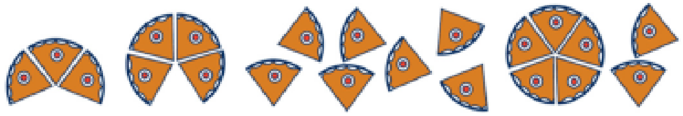
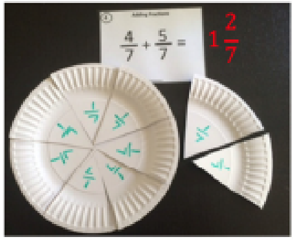
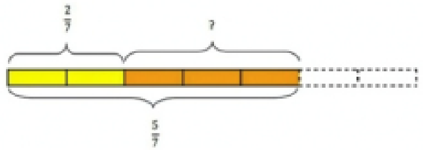
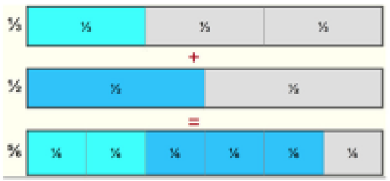
Progression in fractions– Year 4

Concrete	Pictorial	Abstract
Add an subtract fractions with the same denominator  $\frac{2}{6} + \frac{1}{6} = \frac{3}{6}$.	$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$.  =	Hasan eats $\frac{2}{7}$ of his birthday cake. How much does he have left? Jess and Emily both eat $\frac{3}{8}$ of a cake. How much have the eaten in total?
Solve problems involving increasingly harder fractions to calculate quantities and fractions to divide quantities, including non-unit fractions where the answer is a whole $\frac{1}{4}$  $\frac{3}{4}$	 $\frac{1}{4}$ $\frac{3}{4}$	$\frac{3}{4}$ of £16 is £12 £16 ÷ 4 = £4 £4 X 3 = £12

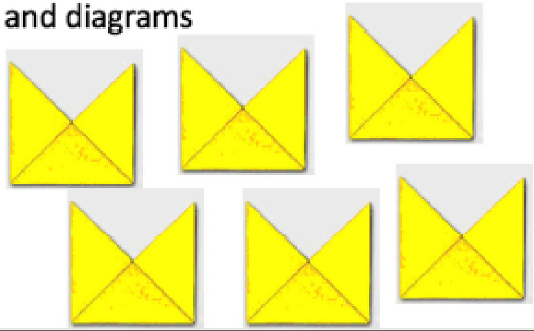
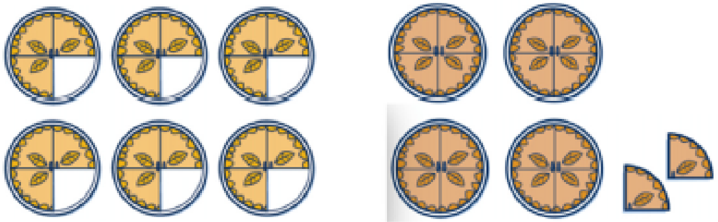
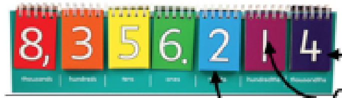
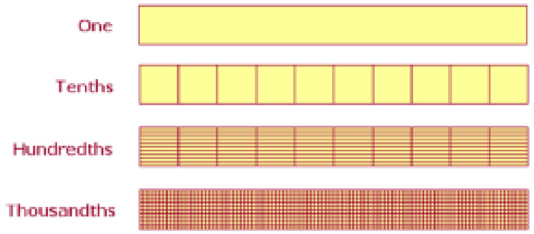
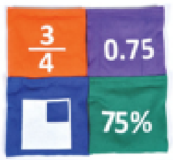
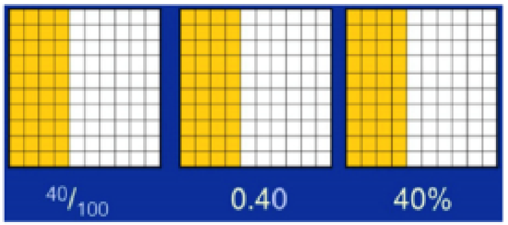
Progression in fractions- Year 5

Concrete	Pictorial	Abstract
Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths  $\frac{6}{8}$ Is equivalent to $\frac{12}{16}$	 $\frac{6}{10} = \frac{60}{100}$	$\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$ $\frac{3}{4} = \frac{75}{100}$ $\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$
Compare and order fractions whose denominators are all multiples of the same number  $\frac{2}{5}$ has become $\frac{8}{20}$ > $\frac{1}{4}$ has become $\frac{5}{20}$		$\frac{2 \times 6}{3 \times 6} = \frac{12}{18}$ $\frac{5 \times 3}{6 \times 3} = \frac{15}{18}$ $\frac{1 \times 6}{3 \times 6} = \frac{6}{18}$ $\frac{2}{5} \xrightarrow{\times 4} = \frac{8}{20} > \frac{1}{4} \xrightarrow{\times 5} = \frac{5}{20}$

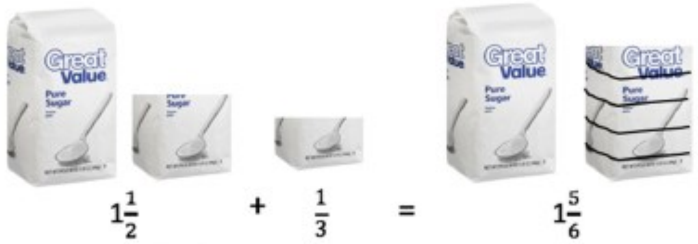
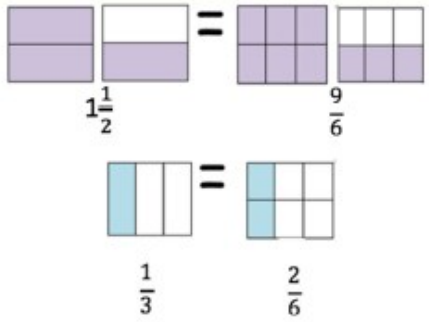
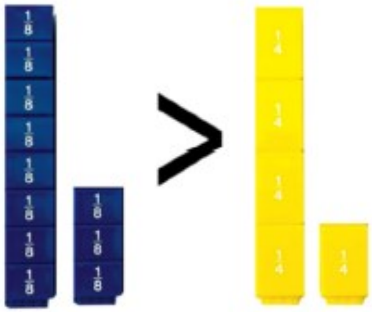
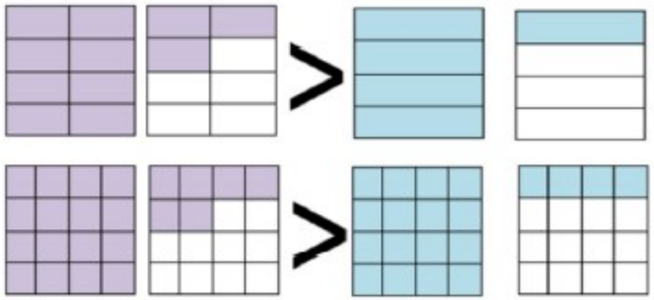
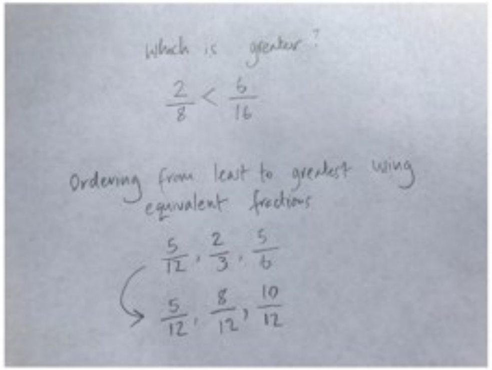
Progression in fractions– Year 5

Concrete	Pictorial	Abstract
Recognise mixed numbers and improper fractions. Convert from on form to the other and write mathematical statements as a mixed number 	 $\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$	$\frac{7}{5} = 1\frac{2}{5}$ because 5 goes into 7 once to make a whole ($\frac{5}{5}$) and $\frac{2}{5}$ are left over
Add and subtract fractions with the same denominators and denominators that are multiples of the same number 	 $\frac{5}{7} - \frac{2}{7} = \boxed{}$ 	$\frac{2}{5} - \frac{1}{4}$ $\frac{2}{5} \overset{\times 4}{=} \frac{8}{20}$ $\quad$ $\frac{1}{4} \overset{\times 5}{=} \frac{5}{20}$ So, $\frac{8}{20} - \frac{5}{20} = \frac{3}{20}$ $\frac{2}{5} - \frac{1}{4} = \frac{3}{20}$

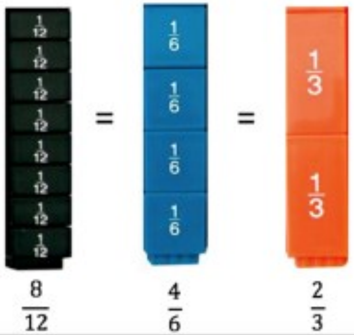
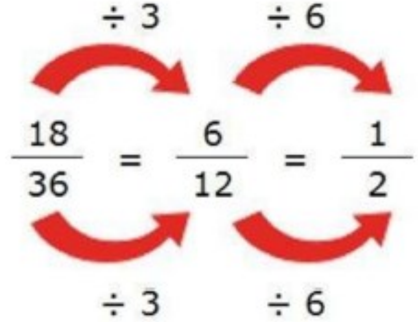
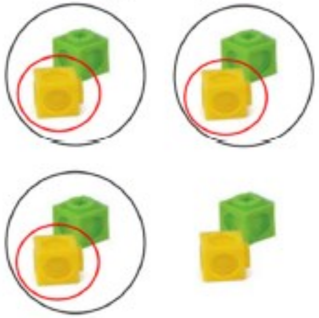
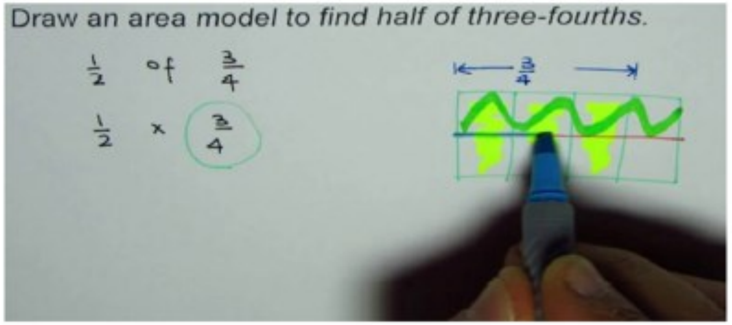
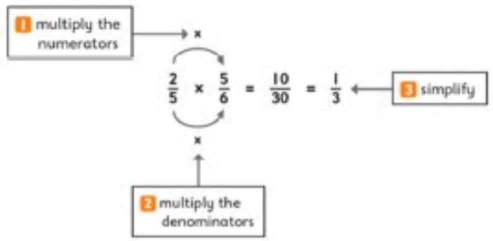
Progression in fractions- Year 5

Concrete	Pictorial	Abstract
Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams  How much paper do we have in total as a fraction/ mixed number?	 6 lots of $\frac{3}{4}$ = $4\frac{2}{4}$	Multiply a proper fraction by a whole number $\frac{3}{4} \times 6 = \frac{18}{4}$ Convert the improper fraction to a mixed number $\frac{18}{4} = 4\frac{2}{4}$
Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents  Four thousandths (0.004) $\frac{4}{1000}$ One hundredth (0.01) $\frac{1}{100}$ Two tenths (0.2) $\frac{2}{10}$		**67.153** How many thousandths does this number have? How many more thousandths do you need to add to make 67.16?
Recognise the % symbol and understand the meaning: write % as a fraction, decimal and percentage 	 	$\frac{4}{10} = 40\% = 0.4$ $\frac{32}{100} = 32\% = 0.32$ $\frac{75}{100} = 75\% = 0.75$ $\frac{2}{25} = \frac{8}{100} = 8\% = 0.08$

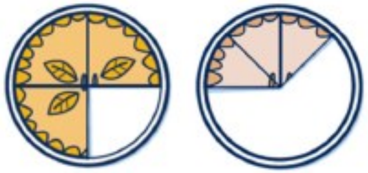
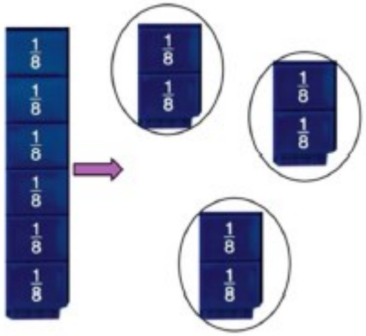
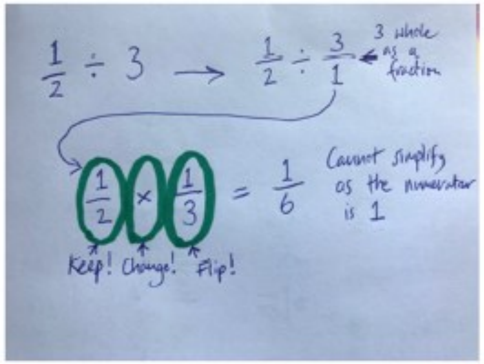
Progression in fractions– Year 6

Concrete	Pictorial	Abstract
Add and subtract fraction with different denominators and mixed numbers using the concept of equivalent fractions 		$1\frac{1}{2} + \frac{1}{3} = 1\frac{5}{6}$ because $1\frac{1}{2} = \frac{3}{2}$ $\frac{3}{2} = \frac{9}{6}$ and $\frac{1}{3} = \frac{2}{6}$ so $\frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1\frac{5}{6}$
Compare and order fractions including fractions > 1 		

Progression in fractions– Year 6

Concrete	Pictorial	Abstract
Use common factors to simplify fractions; use common multiples to express fractions in the same denomination 		
Multiply simple pairs of proper fractions, writing the answer in the simplest form $\frac{1}{2}$ of $\frac{3}{4}$ 		multiply $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ multiply 

Progression in fractions– Year 6

Concrete	Pictorial	Abstract
Recall and use equivalences between simple fractions, decimals and percentages including in different contexts 	Which would you prefer 75% or $\frac{3}{8}$ of a pie?  75% $\quad \frac{3}{8}$	John scored 40/60 in his test and Hannah scored 40%. Who got the greatest score? John = $\frac{40}{60}$ = 66.6% Hannah = 40%
Divide proper fractions by whole numbers $\frac{6}{8}$ divided by 3 = $\frac{2}{8}$ or $\frac{1}{4}$ 	 $\frac{1}{2} \div 3 = \frac{1}{6}$	

<u>Appendix</u>

Mathematical Language